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Chapter - Correlation

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Degree of Correlation	Positive	Negative
Perfect	+1	-1
High	0.75 to 0.99	0.75 to 0.99
Moderate	0.25 to 0.75	0.25 to 0.75
Low	0 to 0.25	0 to 0.25
Absence	0	0

Methods of Correlation

1. Karl Pearson's coefficient of Correlation Method.
2. Spearman's Rank Difference Method.
3. Concurrent Deviation Method.
4. Least Square Method.

1. Karl Pearson's Coefficient of Correlation Method

(A) Direct Method

$$r = \frac{\sum dx dy}{N \cdot \sigma_x \cdot \sigma_y} \quad \text{or} \quad \frac{\sum dx dy}{N \sqrt{\frac{\sum dx^2}{N}} \times \sqrt{\frac{\sum dy^2}{N}}}$$

$$\text{or} \quad \frac{\sum dx dy}{N \cdot \frac{\sqrt{\sum dx^2}}{\sqrt{N}} \cdot \frac{\sqrt{\sum dy^2}}{\sqrt{N}}} \quad \text{or} \quad \frac{\sum dx dy}{\sqrt{\sum dx^2 \sum dy^2}}$$

(B) Indirect Method

$$r = \frac{\sum dx dy - \frac{(\sum dx)(\sum dy)}{N}}{N \cdot \sqrt{\left[\frac{\sum dx^2}{N} - \left(\frac{\sum dx}{N} \right)^2 \right] \left[\frac{\sum dy^2}{N} - \left(\frac{\sum dy}{N} \right)^2 \right]}}$$

$$\text{or} \quad \frac{\sum dx dy \cdot N - (\sum dx \cdot \sum dy)}{[\sum dx^2 \cdot N - (\sum dx)^2] [\sum dy^2 \cdot N - (\sum dy)^2]}$$

Direct Method

Ex:-	X	$n=30$ dx	dx^2	Y	$n=25$ dy	dy^2	$dx dy$
	23	-7	49	18	-7	49	49
	27	-3	9	20	-5	25	15
	28	-2	4	22	-3	9	6
	28	-2	4	27	2	4	4
	29	-1	1	21	-4	16	4
	30	0	0	29	4	16	0
	31	1	1	27	2	4	2
	33	3	9	29	4	16	12
	35	5	25	28	3	9	15
	36	6	36	29	4	16	24
	<u>300</u>	<u>0</u>	<u>138</u>	<u>250</u>	<u>0</u>	<u>164</u>	<u>123</u>

$$\bar{X} = \frac{\sum dx}{N}$$

$$= \frac{300}{10} = 30$$

$$\bar{Y} = \frac{\sum dy}{N}$$

$$= \frac{250}{10} = 25$$

$$r = \frac{\sum dx dy}{\sqrt{\sum dx^2 \times \sum dy^2}}$$

$$= \frac{123}{\sqrt{138 \times 164}}$$

$$= \frac{123}{150.43}$$

$$= +0.82$$

It indicates of high degree in positive correlation.

Indirect Method

X	$\sum dx^2$	$\sum dx^2$	Y	$\sum dy^2$	$\sum dy^2$	$\sum dx dy$
23	-5	25	18	-11	121	55
27	-1	1	20	-9	81	9
28	0	0	22	-7	49	0
28	0	0	27	-2	4	0
29	1	1	21	-8	64	-8
30	2	4	29	0	0	0
31	3	9	27	-2	4	-6
33	5	25	29	0	0	0
35	7	49	28	-1	1	-7
36	8	64	29	0	0	0
	20	178		-40	324	43

$$r = \frac{\sum dx dy}{N} - N \times \frac{\sum dx}{N} \times \frac{\sum dy}{N}$$

$$N \cdot \sqrt{\left[\frac{\sum dx^2}{N} - \left(\frac{\sum dx}{N} \right)^2 \right] \left[\frac{\sum dy^2}{N} - \left(\frac{\sum dy}{N} \right)^2 \right]}$$

$$= 43 - 10 \times \frac{20}{10} \times \left(-\frac{40}{10} \right)$$

$$10 \cdot \sqrt{\left[\frac{178}{10} - \left(\frac{20}{10} \right)^2 \right] \left[\frac{324}{10} - \left(-\frac{40}{10} \right)^2 \right]}$$

$$= \frac{43 + 80}{10 \sqrt{[17.8 - 4] \times [32.4 - 16]}}$$

$$= \frac{123}{10 \sqrt{13.8 \times 16.4}}$$

$$= \frac{123}{10 \times 15.04} = \frac{123}{150.4}$$

$$= +0.82$$

It indicates of high degree - in positive correlation.

Karl Pearson's Method

When Deviations taken from Actual Arithmetic Means

1. Calculate the coefficient of correlation from the following data :

X	:	12	9	8	10	11	13	7
Y	:	14	8	6	9	11	12	3

Ans. $\bar{X} = 10$, $\bar{Y} = 9$, $r = +0.948$

2. Calculate Karl Pearson's coefficient of correlation between X and Y :

X :	45	70	65	30	90	40	50	75	85	60
Y :	35	90	70	40	95	40	60	80	80	50

(B.Com., Meerut 1995)

Ans. $r = +0.903$

Hint : $\bar{x} = 61$, $\bar{y} = 64$

3. Calculate coefficient of correlation between expenditure and sales of following 10 firms :

Firms	:	1	2	3	4	5	6	7	8	9	10
Expenditure	:	11	13	14	16	16	15	15	14	13	13
Sales	:	50	50	55	60	65	65	65	60	60	50

(B.Com., Jabalpur 1991, 94)

Ans. $r = +0.7866$; $\bar{x} = 14$, $\bar{y} = 58$

4. From the following data related to overhead and production cost, calculate the coefficient of correlation by the method of Karl Pearson :

Overheads (Rs. '000)	:	80	90	100	110	120	130	140	150	160
Cost (Rs. '000)	:	15	15	16	19	17	18	16	18	19

Ans. $r = +0.693$; $\bar{x} = 120$, $\bar{y} = 17$

5. Find Karl Pearson's coefficient of correlation from the following figures of heights of fathers and sons :

Height of Fathers (inches)	:	64	65	66	67	68	69	70
Height of sons (inches)	:	66	67	65	68	70	68	72

Ans. $\bar{x} = 67$, $\bar{y} = 68$, $r = +0.81$

When variable are unknown from the actual